# Unit-5 FLOW THROUGH PIPES



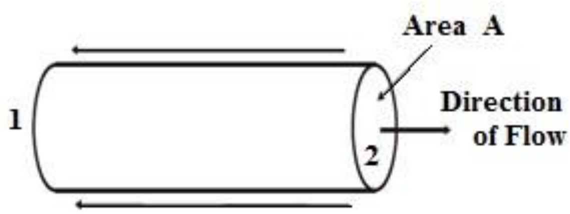
## 5-1 Friction Losses of Head in Pipes:

There are many types of losses of head for flowing liquids such as friction, inlet and outlet losses. The major loss is that due to frictional resistance of the pipe, which depends on the inside roughness of the pipe. The common formula for calculating the loss of head due to friction is Darcy's one.

### *Darcy's formula for friction loss of head:*

For a flowing liquid, water in general, through a pipe, the horizontal forces on water between two sections (1) and (2) are:

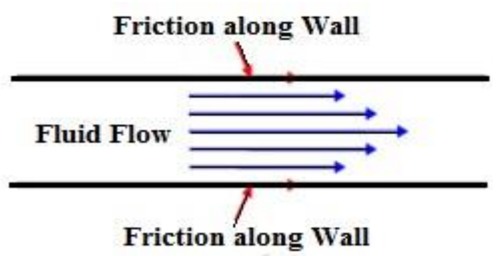
$$P1\ A = P2\ A + FR$$




P1= Pressure intensity at (1).

A = Cross sectional area of pipe. P2= Pressure intensity at (2).

FR= Frictional Resistance at (2).




FR / □ A = (P1 / □) - (P2 / □) = hf

Where, hf = Loss of pressure head due to friction.

□ = Specific gravity of water.

*It is found experimentally that:*

FR = Factor x Wetted Area x $\text{Velocity}^2$

FR = (□ f / 2g) x (□ d L) x $v^2$

Where, f = Friction coefficient.

d = Diameter of pipe.

L = Length of pipe.

hf = (□ f / 2g) x (□ d L) x v2 = 4 f * L * v[1]

□ (□ d2 /4) d * 2 g

$$\mathbf{hf = \frac{4\ f\ L\ v\ 2}{2\ g\ d}}$$

It may be substituted for [v = Q / (□ d2 /4)] in the last equation to get the head loss for a known discharge. Thus,

$$\mathbf{hf = \frac{32\ f\ L\ Q\ 2}{\square 2\ g\ d\ 5}}$$

[1] 2 x 9.81 x 1

***Note:*** In American practice and references, $\lambda = f_{American} = 4f$

**Example 1:**

A pipe 1 m diameter and 15 km long transmits water of velocity of 1 m/sec. The friction coefficient of pipe is 0.005.

Calculate the head loss due to friction?

***Solution***

$$hf = \frac{4\ f\ L\ v\ 2}{2\ g\ d}$$

hf = 4x0.005x15000x 12 = 15.29 m

The Darcy – Weisbach equation relates the head loss (or pressure loss) due to friction along a given length of a pipe to the average velocity of the fluid flow for an incompressible fluid.

The friction coefficient f (or $\lambda = 4f$) is not a constant and depends on the parameters of the pipe and the velocity of the fluid flow, but it is known to high accuracy within certain flow regimes.

For given conditions, it may be evaluated using various empirical or theoretical relations, or it may be obtained from published charts.

$R_e$ (Reynolds Number) is a dimensionless number. $\mathbf{R_e = \frac{\rho\ v\ d}{\mu}}$

*For pipes,* Laminar flow, $R_e < 2000$

Transitional flow, $2000 < R_e < 4000$

Turbulent flow, $R_e > 4000$

*For laminar flow,*

Poiseuille law, ($f = 64/Re$) where Re is the Reynolds number .

*For turbulent flow,*

Methods for finding the friction coefficient f include using a diagram such as the Moody chart, or solving equations such as the Colebrook–White equation.

Also, a variety of empirical equations valid only for certain flow regimes such as the Hazen – Williams equation, which is significantly easier to use in calculations. However, the generality of Darcy – Weisbach equation has made it the preferred one.

The only difference of (hf) between laminar and turbulent flows is the empirical value of (f).

Introducing the concept of smooth and rough pipes, as shown in Moody chart, *we find:*

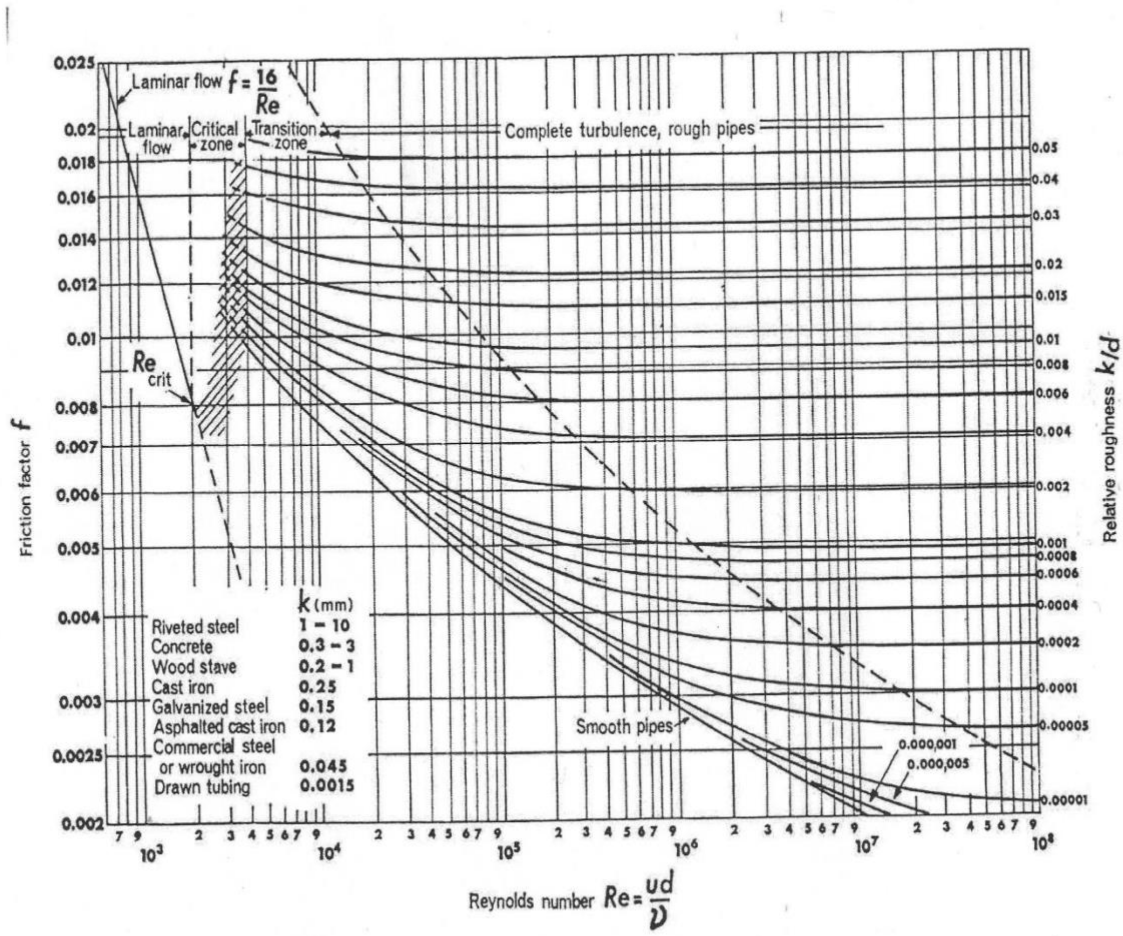
1) For laminar flow, $f = 16 / R_e$
2) For transitional flow, pipes' flow lies outside this region.
3) For smooth turbulent (a limiting line of turbulent flow), all values of relative roughness ($k_s/d$) tend toward this line as R decreases. Blasius equation: $f = 0.079 / R_e^{0.25}$

4) For transitional turbulent, it is the region where (f) varies with both ($k_s$/d) & ($R_e$). Most pipes lie in this region.
5) For rough turbulent, (f) is constant for given ($k_s$/d) and is independent of ($R_e$).

Doing a large number of experiments for the turbulent region for commercial pipes, Colebrook-White established the equation:

$$\frac{1}{\sqrt{f}} = -4\log_{10}\left(\frac{k_s}{3.71d} + \frac{1.26}{\mathrm{Re}\sqrt{f}}\right)$$

This equation is easily solved employing Moody chart.




**Moody Chart**

$\lambda = 4 f$ & values of $k_s$ are provided by pipe manufactures.

| Pipe Material | K, mm |
|---|---|
| Brass, Copper, Glass | 0.003 |
| Asbestos Cement | 0.03 |

| | |
|---|---|
| Iron | 0.06 |
| Galvanised Iron | 0.15 |
| Plastic | 0.03 |
| Bitumen-lined Ductile Iron | 0.03 |
| Concrete-lined Ductile Iron | 0.03 |

**Example 2:**

Water flows in a steel pipe (d = 40 mm, k = $0.045 \times 10^{-3}$ m, μ = 0.001 k/ms) with a rate of 1 lit/s.

Determine the friction coefficient and the head loss due to friction per meter length of the pipe using:

1- Moody chart? 2- Smooth pipe formula?

***Solution***

$v = Q / A = 0.001 / (\pi (0.04)^2/4) = 0.796$ m/s

$R_e = \rho\, v\, d / \mu = (1000 \times 0.796 \times 0.04) / 0.001 = 31840 > 4000$

Turbulent flow.

*1. Moody chart:*

$k/d = 0.045 \times 10^{-3} / 0.04 = 0.0011$ & $Re = 31840$

from the chart, $f = 0.0065$

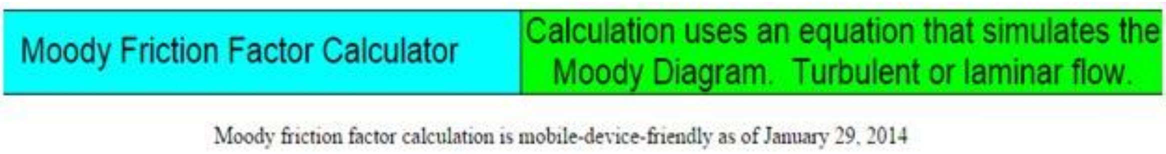
$$hf = \frac{4\,f\,L\,v^2}{2\,g\,d} = \frac{4x0.0065x1x(0.796)2}{2x9.81x0.04} = 0.0209 \text{ m / m of pipe}$$

*2. Smooth pipe (Blasius equation):*

$f = 0.079 / R_e^{0.25} = 0.079 / (31840) = 0.0059$

$$hf = \frac{4\,f\,L\,v^2}{2\,g\,d} = \frac{4x0.0059x1x(0.796)2}{2x9.81x0.04} = 0.02 \text{ m / m of pipe}$$

**Another Solution:**



**Select Calculation:**

- (•) Circular Duct: Enter D and Q
- ( ) Circular Duct: Enter D and V
- ( ) Circular Duct: Enter D and Re
- ( ) Non-circular Duct: Enter A, P, and Q
- ( ) Non-circular Duct: Enter A, P, and V
- ( ) Non-circular Duct: Enter A, P, and Re



Initial Values

Click to Calculate

| | |
|---|---|
| Kinematic viscosity, ν ($L^2/T$): | 1.0E-6 |
| Surface Roughness, e (L): | 4.5E-5 |
| Duct Diameter, D (L): | 0.04 |
| Duct Area, A ($L^2$): | 0.0012566371 |
| Duct Perimeter, P (L): | 0.12566371 |
| Velocity, V (L/T): | 0.79577472 |
| Discharge, Q ($L^3/T$): | 0.001 |
| Reynolds Number: | 31830.989 |
| e/D: | 0.001125 |
| Moody Friction Factor, f: | 0.026171935 |

$$f = \frac{64}{\mathrm{Re}} \quad \text{for } \mathrm{Re} \leqslant 2100 \text{(laminar flow)} \qquad \mathrm{Re} = \frac{VD}{\nu}$$

$$f = \frac{1.325}{\left[\ln\left(\frac{e}{3.7\mathrm{D}} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \quad \text{for } 5000 \leqslant \mathrm{Re} \leqslant 10^8 \text{(turbulent flow) and } 10^{-6} \leqslant \frac{e}{D} \leqslant 10^{-2}$$

D = Diameter of a circular duct. If duct is non-circular, then D is computed as the hydraulic diameter of a rectangular duct, where D = 4A / P per our non-circular duct page.
Re = Reynolds Number. The symbol Re is not the product (R)(e).

Kinematic viscosity (ν) depends on the fluid (water, air, etc.). Click for table.
Surface roughness depends on the duct material (steel, plastic, iron, etc.). Click for table.

The equations used in this program represent the Moody diagram which is the old-fashioned way of finding f. You may enter numbers in any units, so long as you are consistent. (L) means that the variable has units of length (e.g. meters). ($L^3/T$) means that the variable has units of cubic length per time (e.g. $m^3/s$). The Moody friction factor (f) is used in the Darcy-Weisbach major loss equation. Note that for laminar flow, f is independent of e. However, you must still enter an e for the program to run even though e is not used to compute f. Equations can be found in Discussion and References for Closed Conduit Flow.

A more complicated equation which represents a slightly larger range of Reynolds numbers and e/D's is used in Design of Circular Liquid or Gas Pipes.



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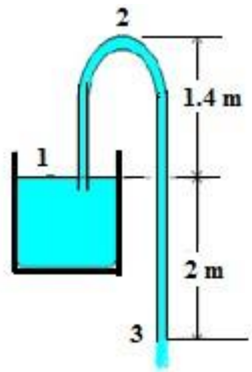
August 25, 2015: Made text fields show 8 significant figures rather than 16. Calculation still uses double precision internally.

**Example 3:**

The pipe of a syphon has 75 mm diameter and discharges water to the atmosphere, as shown in figure.
Neglect all possible losses.

a. Determine the velocity of flow?
b. Find the discharge?
c. What is the absolute pressure at the point 2?




***Solution***

**(a)** *Applying Bernoulli's equation between (1) and (3),* $2 + 0 + 0 = 0 + 0 + (v^2_3/2g)$

$$v_3 = 6.26 \text{ m/s}$$

**(b)** $Q = v_3 \times A = 6.26 \times (\pi\,(0.075)^2/4) = 0.028 \text{ m}^3/\text{s}$

**(c)** *Applying Bernoulli's equation between (1) and (2),*

$$2 + 0 + 0 = 3.4 + P_2/\rho g + (6.26^2/2g)$$

$$P_2 = -\,3.397 \times (1000 \times 9.81) = -\,33327.8 \text{ N/m}^2 = -\,33.33 \text{ kPa}$$

$P_{2abs} = 64.77$ kPa *where,* ($P_{atm} = 98.1$ kN/m$^2$)

## 5-2 Secondary Losses of Head in Pipes:

Any change in a pipe (in direction, in diameter, having a valve or other fitting) will cause a loss of energy due to the disturbance in the flow.

$$h_s = K\,(v^2 / 2g)$$

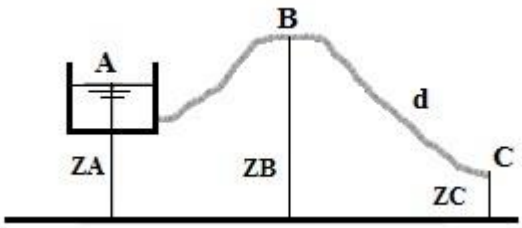
The velocity v is the velocity at the entry to the fitting. When the velocity changes upstream and downstream the section, the larger velocity is generally used.

| **Obstruction** | **K** |
|---|---|
| Tank Exit | 0.5 |
| Tank Entry | 1.0 |
| Smooth Bend | 0.3 |
| 90º Elbow | 0.9 |
| 45º Elbow | 0.4 |
| Standard T | 1.8 |
| Strainer | 2.0 |
| Angle Valve, wide open | 5.0 |
| *Gate Valve:* | 0.2 |
| Wide Open | |
| 3/4 open | 1.2 |
| 1/2 open | 5.6 |

| | |
|---|---|
| 1/4 open | 24.0 |
| Sudden Enlargement | 0.1 |
| *Sudden Contraction:* | |
| Area Ratio ($A_2/A_1$) = 0.2 | 0.4 |
| Area Ratio ($A_2/A_1$) = 0.4 | 0.3 |
| Area Ratio ($A_2/A_1$) = 0.6 | 0.2 |
| Area Ratio ($A_2/A_1$) = 0.7 | 0.1 |

**Example 4:**

A pipe transmits water from a tank A to point C that is lower than water level in the tank by 4 m. The pipe is 100 mm diameter and 15 m long.




The highest point on the pipe B is 1.5 m above water level in the tank and 5 m long from the tank. The friction factor (4 f) is 0.08, with sharp inlet and outlet to the pipe.

a. Determine the velocity of water leaving the pipe at C?
b. Calculate the pressure in the pipe at the point B?

***Solution***

**(a)** *Applying Bernoulli's equation between A and C,*

Head loss due to entry (tank exit, *from table*) = 0.5 ($v^2_C/2g$)

Head loss due to exit into air without contraction = 0

$$Z_A + 0 + 0 = Z_C + 0 + (v^2_C/2g) + 0.5\,(v^2_C/2g) + 0 + \frac{4\,f\,L\;v^2_C}{2\,g\,d}$$

$4 = (v^2_C/2g) \times \{1 + 0.5 + (4x0.08x15)/0.1\}$

$v_C = 1.26$ m/s

**(b)** *Applying Bernoulli's equation between A and B,*

$$Z_A + 0 + 0 = Z_B + P_B/\rho g + (v_B^2/2g) + 0.5\,(v^2_B/2g) + \frac{4\,f\,L\;v^2_B}{2\,g\,d}$$

$- 1.5 = P_B/(1000x9.81) + (1.26^2/2x9.81) * \{1 + 0.5 + (4x0.08x5)/0.1\}$

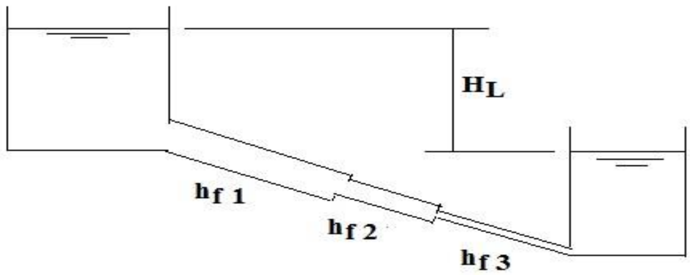
$P_B = - 28.61$ kN/m$^2$

## 5-3 Flow through Pipe Systems:

***Pipes in Series:***

Pipes in series are pipes with different diameters and lengths connected together forming a pipe line. Consider pipes in series discharging water from a tank with higher water level to another with lower water level, as shown in the figure.

Neglecting secondary losses, it is obvious that the total head loss HL between the two tanks is the sum of the friction losses through the pipe line.




Friction losses through the pipe line are the sum of friction loss of each pipe. **HL = hf 1 + hf 2 + hf 3 + .....**

$$HL = \frac{4f_1L_1v_1^2}{2gd_1} + \frac{4f_2L_2v_2^2}{2gd_2} + \frac{4f_3L_3v_3^2}{2gd_3} + \dots$$

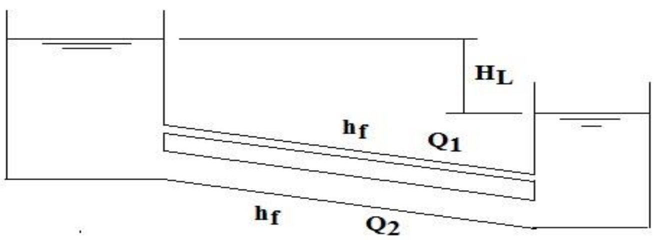
*OR:*

$$HL = \frac{32f_1L_1Q^2}{\pi^2 g d_1^5} + \frac{32f_2L_2Q^2}{\pi^2 g d_2^5} + \frac{32f_3L_3Q^2}{\pi^2 g d_3^5} + \dots$$

## ***Pipes in Parallel:***

Pipes in parallel are pipes with different diameters and same lengths, where each pipe is connected separately to increase the discharge. Consider pipes in parallel discharging water from a tank with higher water level to another with lower water level, as shown in the figure.

Neglecting minor losses, it is obvious that the total head loss HL between the two tanks is the same as the friction losses through each pipe.



The friction losses through all pipes are the same, and all pipes discharge water independently.

**$HL = h_{f1} = h_{f2} = .....$**

$L_1 = L_2 = L$

$$HL = \frac{4 f_1 L v_1^2}{2 g d_1} = \frac{4 f_2 L v_2^2}{2 g d_2} = .....$$

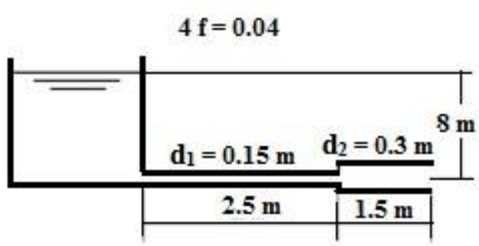
$$HL = \frac{32 f_1 L Q_1^2}{\pi^2 g d_1^5} = \frac{32 f_2 L Q_2^2}{\pi^2 g d_2^5} = .....$$

$$Q = Q_1 + Q_2$$

**<u>Example 5:</u>**

A pipe, 40 m long, is connected to a water tank at one end and flows freely in atmosphere at the other end. The diameter of pipe is 15 cm for first 25 m from the tank, and then the diameter is suddenly enlarged to 30 cm. Height of water in the tank is 8 m above the centre of pipe. Darcy's coefficient is 0.01. Determine the discharge neglecting minor losses?

***<u>Solution</u>***




Loss due to friction, $h_{Lf} = h_{f1} + h_{f2}$

$$h_f = \frac{32\ f\ L\ Q^2}{\square 2\ g\ d\ 5} \qquad f = 0.01$$

Total losses, $$h_T = Q^2 \left( \frac{32\ f\ L_1}{\square^2\ g d_1^5} + \frac{32\ f\ L_2}{\square^2\ g d_2^5} \right)$$

$$8 = Q^2 \left( \frac{(32 x 0.01)\ x\ (25)}{\square^2\ g\ (0.15)^5} + \frac{(32 x 0.01)\ (15)}{\square^2\ g\ (0.3)^5} \right)$$

$\square Q = 0.087\ m^3/sec$

**Example 6:**

Two pipes are connected in parallel between two reservoirs that have difference in levels of 3.5 m. The length, the diameter, and friction factor (4 f) are 2400 m, 1.2 m, and 0.026 for the first pipe and 2400 m, 1 m, and 0.019 for the second pipe.

Calculate the total discharge between the two reservoirs?

***Solution***

$$HL = \frac{32\, f_1\, L\, Q_1^2}{\pi^2\, g\, d_1^5} = \frac{32\, f_2\, L\, Q_2^2}{\pi^2\, g\, d_2^5}$$

$$3.5 = \frac{32\, f_1\, L\, Q_1^2}{\pi^2\, g\, d_1^5} = \frac{8 \times 0.026 \times 2400 \times Q_1^2}{\pi^2 \times 9.81 \times 1.2^5}$$

$$Q_1 = 1.29 \text{ m}^3/\text{sec}$$

$$3.5 = \frac{32\, f_2\, L\, Q_2^2}{\pi^2\, g\, d_2^5} = \frac{8 \times 0.019 \times 2400 \times Q_2^2}{\pi^2 \times 9.81 \times 1^5}$$

$$Q_2 = 0.96 \text{ m}^3/\text{sec}$$

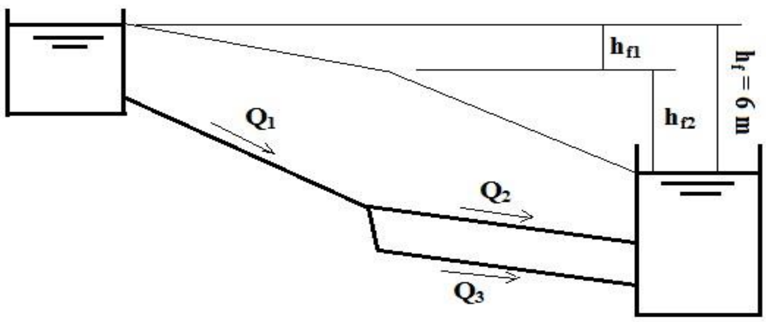
$$\therefore Q = Q_1 + Q_2 = 1.29 + 0.96 = 2.25 \text{ m}^3/\text{sec}$$

**Example**

**7:**

Two reservoirs have 6 m difference in water levels, and are connected by a pipe 60 cm diameter and 3000 m long. Then, the pipe branches into two pipes each 30 cm diameter and 1500 m long. The friction coefficient is 0.01.

Neglecting minor losses, determine the flow rates in the pipe system?




**Solution**

$h_f = h_{f1} + h_{f2}$

$6 = h_{f1} + h_{f2}$

$6 = k_1 Q_1^2 + k_2 Q_2^2$

$$k_1 = \frac{32\, f1\, L1}{\square 2\, g\, d15} = \frac{32*0.01*3000}{\square 2*9.81*0.65} = 127.64$$

$$k_2 = \frac{32\, f2\, L2}{\square 2\, g\, d2\ 5} = \frac{32*0.01*1500}{\square 2*9.81*0.35} = 4084.48$$

**Example**



$k_2 = 32\ k_1$

 $6 = k_1\ Q_1^2 + 32\ k_1\ Q_2^2$

$h_{f2} = h_{f3}$ & $k_2 = k_3$  $Q_2 = Q_3$

$Q_1 = Q_2 + Q_3 = 2\ Q_2$

 $6 = k_1\ Q_1^2 + 8\ k_1\ Q_1^2 = 9\ k_1\ Q_1^2 = (9 * 127.64)\ Q_1^2 = 1148.76\ Q_1^2$

 $Q_1 = 0.072\ m^3/s$

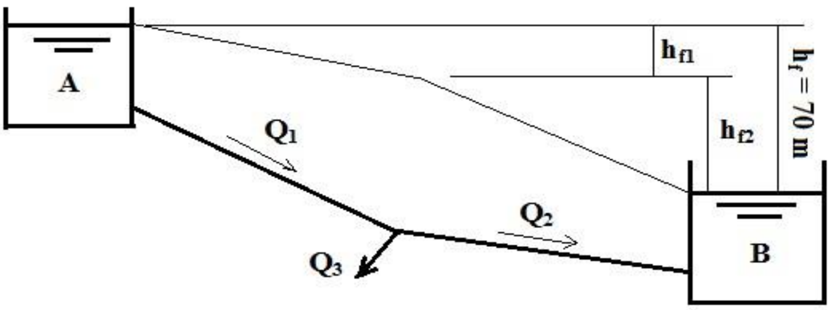
& $Q_2 = 0.036\ m^3/s$

**8:**

Two tanks A and B have 70 m difference in water levels, and are connected by a pipe 0.25 m diameter and 6 km long with 0.002 friction coefficient. The pipe is tapped at its mid point to leak out 0.04 $m^3/s$ flow rate. Minor losses are ignored.

Determine the discharge leaving tank A?
Find the discharge entering tank B?

**Solution**




$h_f = h_{f1} + h_{f2}$

$70 = h_{f1} + h_{f2}$

**Example**



$70 = k_1 Q_1^2 + k_2 Q_2^2$

$k_1 = k_2 = \dfrac{32\ f\ L}{\square 2\ g\ d5} = \dfrac{32*0.002*3000}{\square 2*9.81*0.255} = 2032.7$

□ $70 = k_1 Q_1^2 + k_1 Q_2^2$

$Q_1 = Q_2 + Q_3 = Q_2 + 0.04$

□ $70 = k_1 (Q_2 + 0.04)^2 + k_1 Q_2^2$

$= k_1 (Q_2^2 + 0.08\ Q_2 + 0.0016) + k_1 Q_2^2$

$= k_1 Q_2^2 + 0.08\ k_1 Q_2 + 0.0016\ k_1 + k_1 Q_2^2$

$= 2\ k_1 Q_2^2 + 0.08\ k_1 Q_2 + 0.0016\ k_1$

$= 4065.4\ Q_2^2 + 162.6\ Q_2 + 3.25$

$0.0172 = Q_2^2 + 0.04\ Q_2 + 0.0008$

$Q_2^2 + 0.04\ Q_2 - 0.0164 = 0$

$$Q2 = \frac{-0.04 \pm \sqrt{(-0.04)^2 - 4(1)(-0.0164)}}{2(1)}$$

□ $Q_2 = 0.11\ m^3/s$ & $Q_1 = 0.15\ m^3/s$

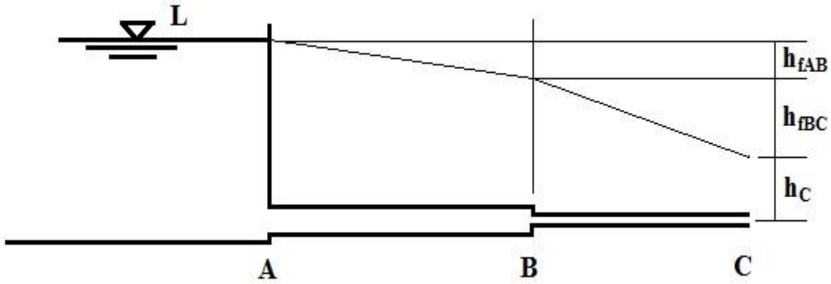
**9:**

A tank transmits 100 L/s of water to the point C where the pressure is maintained at 1.5 kg/cm$^2$. The first part AB of the pipe line is 50 cm diameter and 2.5 km long, and the second part BC is 25 cm diameter and 1.5 km long. The friction coefficient is 0.005 and minor losses are ignored.

**Example**

Assuming level at C is (0.0); find the water level (L) in the tank?

**Solution**




$h_C = P_C / _{\gamma} = 1500 / 1 = 1500$ cm = 15 m

$h_C = 15 = L – h_{fAB} - h_{fBC}$

$$h_{fAB} = \frac{32\ f1\ L1}{\square 2\ g\ d15} = \frac{32*0.005*2500}{\square 2*9.81*0.55} = 1.32$$

$$h_{fBC} = \frac{32\ f2\ L2}{\square 2\ g\ d2\ 5} = \frac{32*0.005*1500}{\square 2*9.81*0.255} = 25.38$$

$15 = L – 1.32 – 25.38$

**Example**



 L = 41.7 m

**Example**

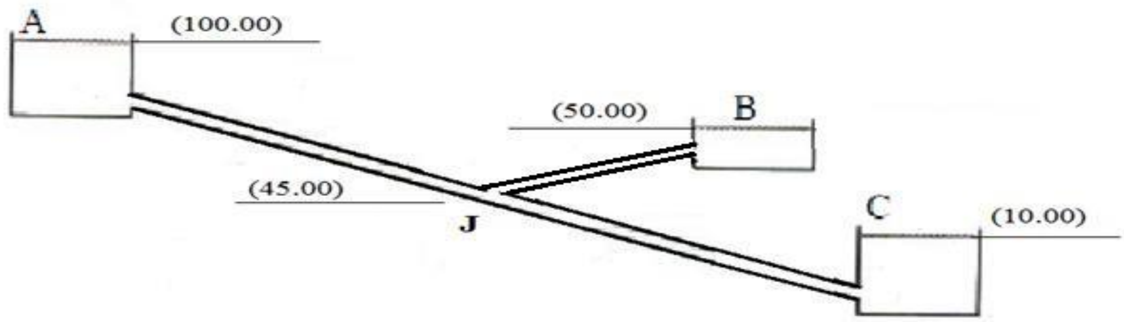


**10:**

Three water tanks A, B and C with water surface levels (100.00), (50.00) and (10.00) m are connected by pipes AJ, BJ and CJ to a common joint J of a level (45.00) m. The three pipes have the same length, diameter and friction coefficient.

a) Calculate the head at the joint J?
b) Determine the discharge in each pipe?

***Solution***




*Assume,* QAJ = QJB + QJC

Applying Bernoulli's equation between A and J:

HA = HJ + hf AJ

100 + 0 + 0 = HJ + hf AJ

100 - HJ = hf AJ = K Q2AJ

*where,* K = 32 f l / □2 g d5

$$Q_{AJ} = (100 - H_J)^{1/2} / (K)^{1/2} \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

Similarly, applying Bernoulli’s equation between J and B:

$$H_J = H_B + hf_{JB}$$

$$H_J - 50 = hf_{JB} = K\, Q^2_{JB}$$

$$Q_{JB} = (H_J - 50)^{1/2} / (K)^{1/2} \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

Also, applying Bernoulli’s equation between J and C:

$$H_J = H_C + hf_{JC}$$

$$H_J - 10 = hf_{JC} = K\, Q^2_{JC}$$

$$Q_{JC} = (H_J - 10)^{1/2} / (K)^{1/2} \quad \ldots\ldots\ldots\ldots\ldots \quad (3)$$

Solving equations 1, 2 and 3 by trial and error, we get:

| **Assumed HJ** | **QAJ x** (K)1/2 | **JB x** (K)1/2 | **QJC x** (K)1/2 | **(QJB+QJC)x**(K)1/2 |
|---|---|---|---|---|
| 70 | 5.48 | 4.47 | 7.745 | 12.216 |
| 60 | 6.325 | 3.162 | 7.07 | 10.233 |
| 53 | 6.855 | 1.732 | 6.557 | 8.289 |
| 51 | 7 | 1 | 6.4 | 7.4 |

| | | | | |
|---|---|---|---|---|
| 50.5 | 7.036 | 0.707 | 6.364 | 7.07 |
| 50.45 | **7.039** | 0.671 | 6.36 | **7.031** |
| 50.4 | 7.043 | 0.632 | 6.356 | 6.988 |
| | | | | |
| 50 | 7.071 | **0** | 6.324 | 6.324 |

From the table:

HJ = 50.45 m

QAJ = 7.039 / (K)1/2

QJB = 0.671 / (K)1/2

QJC = 6.36 / (K)1/2

It has to be noted that if HJ < 50, then the flow will be from B to J.

**Exercise:**

Three water tanks A, B and C are connected to a joint J by three pipes AJ, BJ and CJ such that the water level in tank A is 40 m higher than tank B and 55 m higher than tank C. Each pipe is 1500 m long, 0.3 m diameter and f = 0.01.

Calculate the discharges and directions of flow?

***Solution***

Taking the water level in the tank C as a datum, *the results are:*

HJ = 18 m

QAJ = 0.134 m3/sec

QJB = 0.038 m3/sec

QJC = 0.094 m3/sec